Poincaré polynomials and invariant theory

14 November, 2024

1. Draw the lattice of flats of the hyperplane arrangement in $\mathbb{C}^4$ with

$$Q(\mathcal{A}) = (x+y)(x+z)(x+w)(y+z)(y+w)(z+w).$$

2. Use Polya's theorem to count the number of ways to color the corners of a square with 2 colors, up to
   (a) cyclic symmetry and,
   (b) dihedral symmetry.

3. Use the Reynolds operator to find an invariant element in degrees $4, 5, 6$ of the polynomial ring $\mathbb{Q}[x_1, x_2, x_3]$ acted upon by the symmetric group S_3.

4. Compute the following plethora of plethyms. Compare to your answers for Item 2. What do you notice?
   (a) $(\frac{1}{4}p_{1,1,1,1} + \frac{1}{4}p_{2,2} + \frac{1}{2}p_4)[1+t]$
   (b) $(\frac{1}{8}p_{1,1,1,1} + \frac{1}{4}p_{2,1,1} + \frac{3}{8}p_{2,2} + \frac{1}{4}p_4)[1+t]$